∩AAAA

∪bbbbb

Assuming that a = (A\B)\C, b = (B\A)\C, c = (C\A)\B, d = (A∩B)\C, e = (B∩C)\A, f = (A∩C)\B, g = A∩B∩C

So, A = a∪d∪g∪f, B = b∪d∪g∪e, C = c∪e∪g∪f

According to the definition of symmetric difference, AΔB = a∪f∪b∪e, BΔC = b∪d∪c∪f, AΔC = a∪d∪c∪e.

1. AΔ(BΔC) = AΔ(b∪d∪c∪f) = (a∪d∪f∪g)Δ(b∪d∪c∪f) = a∪b∪c∪g

(AΔB)ΔC = (a∪f∪b∪e)ΔC = (a∪f∪b∪e)Δ(c∪e∪g∪f) = a∪b∪c∪g

Thus, AΔ(BΔC) = (AΔB)ΔC, the operation Δ is associative

1. A¯ = b∪e∪c, B¯ = a∪f∪c, C¯ = a∪d∪b

A∩B∩C = g, A∩B¯∩C¯ = a, B∩C¯∩A¯ = b, C∩B¯∩A¯ = c, A∩B∩C¯ = d, B∩C∩A¯ = e, C∩A∩B¯ = f

(A∩B∩C)∪(A∩B¯∩C¯)∪(B∩C¯∩A¯)∪(C∩B¯∩A¯)∪(A∩B∩C¯)∪(B∩C∩A¯)∪(C∩A∩B¯)=g+a+b+c+d+e+f

A∪B∪C = a+b+c+d+e+f+g

Thus, A∪B∪C=(A∩B∩C)∪(A∩B¯∩C¯)∪(B∩C¯∩A¯)∪(C∩B¯∩A¯)∪(A∩B∩C¯)∪(B∩C∩A¯)∪(C∩A∩B¯)

1. Convert the logic into English: If A Δ (B Δ C)= ∅, then A∪B∪C=((A∩B)∖C)∪((A∩C)∖B)∪((B∩C)∖A)

Given that AΔ(BΔC)=∅, it can be inferred that a∪b∪c∪g =∅, a=b=c=g=∅

A∪B∪C = a∪b∪c∪d∪e∪f∪g = d∪f∪e

((A∩B)∖C)∪((A∩C)∖B)∪((B∩C)∖A) = ((d∪g)∖(c∪f∪g∪e))∪((g∪f)∖(b∪d∪g∪e))∪((g∪e)∖(a∪d∪g∪f))

= d∪f∪e

Thus, A∪B∪C=((A∩B)∖C)∪((A∩C)∖B)∪((B∩C)∖A) when A Δ (B Δ C)= ∅

Assume that a students founds only oyster mushrooms, b students found only maitake mushrooms, c students found only lion’s mane mushrooms, d+g students found both oyster and maitake mushrooms, g+f students found both maitake and lion’s mane mushrooms, g+e students found both oyster and lion’s mane mushrooms and g students had found all three types of mushrooms, x students found nothing.

Then we could have equations:

a+b+c+d+e+f+g+x = 150

a+d+e+g = 100

b+d+g+f = 33

g+e+f+c = 23

d+g = 23

g+f = 12

g+e = 10

g = 2

After calculation we can get that a = 69, b =0, c = 3 and x = 37

1. 69 students found only oyster mushrooms.
2. No student found only maitake mushrooms.
3. 3 students found only lion’s mane mushrooms.
4. 37 students found no mushrooms.

III.

1. True. Because for every element in A, there must be one and only one image in B, the number of elements in set B that has a preimage is the same as the number of elements in set A. Since function g is injective, indicating that all elements in B which has a preimage in function f has one and only one image in set A. Although g(f(x)) might be another element in A, not necessarily x itself, the image set of g(f(x)) still equal to A.
2. False. The result of B×A is a set of all ordered pairs (g(x),f(y)). However, the results of A×B is a set of all ordered pairs (f(x),g(y)), which is not the same as (g(x),f(y)).
3. False. For example, A=(a1,a2,a3), B=(b1,b2,b3), f(a1)=b3, f(a2)=b2, f(a3)=b1 and g(b1)=a1, g(b2)=a3, g(b3)=a2.

f(A) × g(B)=(b3,b2,b1) × (a1,a3,a2), which has nine elements while (f(x),g(y)) has only three elements.